Automatic Control II Case Study: Shaker

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Last Revised 09:00 3 August 2009

Executive Summary

Electromagnetic devices are pervasive in the modern world, seen in motors, solenoids, loudspeakers and shakers. These devices are normally comprised of a permanent DC magnet and a coil. When a current is passed through the coil, a magnetic field is induced which induces a (Lorentz) force on the permanent magnet. In the case of a shaker, this force can be used to drive a structure.

Shakers are commonly found in many gaming and audio devices, and is a common tool used in all dynamics labs where they are used for many applications such as model testing to excite structures, and also active vibration control. There are two main types of electromagnetic shaker; the moving coil shaker and the moving mass shaker.

The moving coil shaker, is very much like a loudspeaker, where the coil moves and the magnet remains fixed. The moving coil can generate a force to act directly on a structure. These shakers have very low diaphragm stiffness and have the advantage that they can generate forces over a large frequency range. The disadvantage of these shakers is that they need a large (high mechanical impedance) structure on which to react against.

The other type of shaker is the moving mass (or magnet) shaker, also known as a proof mass shaker (see Figure 1). These shakers are commonly used in “vibration backpacks” and bass boost audio systems (see Figure 2), as well as active vibration control systems. These shakers work by accelerating the mass, which generates an equal and opposite force on the housing, which drives the structure on to which it is attached. The disadvantage of these shakers is that at low frequencies they generate almost no external reaction force as the Lorentz force simply overcomes the stiffness of the diaphragm (see Figure 5). In addition, proof mass shakers often exhibit a strong resonant peak.

Your task will be to

(a) initially model the mechanical system only. You will learn how to enter state space systems in Matlab, calculate the poles of a system, and from this determine the natural frequency and damping ratio of a second order system. You will learn how to plot the step response and frequency response of a system.

(b) model the fully coupled electro-mechanical system, and a reduced order system. You will investigate if it is possible to control a system given a certain actuator arrangement.

(c) derive a controller using pole placement to improvement the shaker bandwidth and reduce “quality factor” of the resonance. You will learn how to convert a state space model to a transfer function in the Laplace domain, how to calculate the zeros of a state space system, will investigate if it is possible to estimate the states of a system given a certain sensor arrangement, and how to add sensors to a system.
Introduction

The proof mass shaker shown in Figure 1 is the same as those found in gaming backpacks and “bass-boost” shakers used in car audio systems (see Figure 2). The shakers are comprised of an outer coil, which when a current is passed through, induces a magnetic field. The inner part of the shaker is a magnetic proof mass, which is moved by the magnetic field from the coil. The proof mass is held in place by a fibreglass diaphragm which acts as a spring. The shakers have a nominal resonance of 46Hz and are lightly damped (with a quality factor of approximately 10). A power amplifier, with a gain of unity and a bandwidth of 10-200Hz, is used to amplify the input voltages to the plant and supply adequate current to drive the coils.

Figure 2: Photograph of Aura shakers for bass boost in car audio systems
Mathematical Modelling

Before deriving the dynamics of the system a number of assumptions must be made. These are:

- The system is linear. This assumption is true for small displacements (<2mm) of the proof masses.
- Damping is limited to viscous losses and back EMF only. There are no structural (hysteretic) losses or Coulomb damping.
- The base to which the shaker is attached is assumed to be rigid.

![Figure 3: Schematic of the shaker system](image)

The equations of motion of the system may be derived by considering the schematic of the spring-mass-damper system in Figure 3. The force \( f \) is induced by the current and acts equally on both the coil and mass (in opposing directions). With reference to Figure 3 we will make a few definitions to aid us in the derivation of the dynamics.

**Fixed Parameters of Mechanical Subsystem**

- \( m = 0.400 \text{kg} \) is the mass of the magnet,
- \( k = 33.5 \text{kN/m} \) is the stiffness of the diaphragm,
- \( b = 7.78 \text{N/(m/s)} \) is the viscous loss coefficient (which is a result of structural losses in the diaphragm and viscous shear losses from pumping air, both in the air gap between the coil and magnet, but also the piston like motion of the magnet itself which drives the surrounding air). It does not include the back EMF generated by the motion of the magnet past the coil.

**Fixed Parameters of Electrical System**

- \( R_m = 3.80 \Omega \) is the electrical resistance of the coil,
- \( K_m = 3.80 \text{N/A} \) is the electromotive force constant of the coil, and relates the force generated by the coil per unit current in the coil, and is equal to the back EMF constant for SI units,
- \( L_m = 0.5 \text{mH} \) is the electrical inductance of the coil.

**Variables**

- \( x \) (m) is the displacement of the mass. Positive displacements are upwards.
- \( V_{in} \) (V) is the voltage input into the power amplifier used to drive the coil,
- \( V_{out} \) (V) is the voltage output from the power amplifier and input into the coil,
- \( i \) (A) is the current in the coil,
- \( f \) (N) is the force generated by the coils on the mass, where an upwards direction is positive.
**PART A**

Let us first derive the equations of the mechanical sub-system.

**Mechanical system differential equation**

First consider the forces acting on the mass, and are given by

\[ m \dddot{x} + b \ddot{x} + kx = f \]  

(1)

Note that the **natural frequency** of the mechanical sub-system is given by the eigenvalues of the root of the stiffness matrix divided by the mass matrix, i.e.

\[ \omega_n = \frac{k}{\sqrt{m}} = 46 \text{Hz} \]

In state space form the equations of motion given by Equation (1) may be written as

\[
\begin{bmatrix}
\dot{x} \\
\ddot{x}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-k/m & -b/m
\end{bmatrix}
\begin{bmatrix}
x \\
\dot{x}
\end{bmatrix} +
\begin{bmatrix}
0 \\
1/m
\end{bmatrix} f
\]  

(2)

The output of the system is the reaction force generated by the acceleration of the mass arising from the connected mechanical elements (spring and damper) and the force induced by the coil, i.e.

\[
y = f_{\text{reaction}} = m \ddot{x}
\]

\[
= m \begin{bmatrix}
-k/m & -b/m
\end{bmatrix} \begin{bmatrix}
x \\
\dot{x}
\end{bmatrix} + m \begin{bmatrix}
1/m
\end{bmatrix} f
\]

\[
= \begin{bmatrix}
k & b
\end{bmatrix} \begin{bmatrix}
x \\
\dot{x}
\end{bmatrix} + \begin{bmatrix}
1
\end{bmatrix} f
\]  

(3)

**Questions**

First build a state space model of the mechanical system using the code below. You need to fill in a few rows which are marked in **bold**. Create an “m-file” script, then press either “Save and Run”, or even better “Save and Publish to HTML”.

---

```matlab
clc         % Clear the screen
close all   % Close all existing figures
clear all   % Clear all variables from the workspace

%% *** PART A ***
% Covers the fundamentals of modelling a simple mechanical system in
% Matlab

%% Revision History
% Author  Date        Changes
% BSC     29/6/09     Code created
% BSC     30/6/09     Code created
% BSC     1/7/09      Added LaTex mathematical expressions. These are indicated by double $$ signs

% This file has been written with *publishing to html* in mind. Click on
% File > Save and Publish to HTML
```

---
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%% Define the numeric values of the parameters

% Mechanical Parameters
m = 0.400                           % Proof mass (kg)
k = 33500                           % Stiffness of diaphragm (N/m)
b = 7.78                            % Mechanical damping constant (Ns/m)

% Electrical parameters
Rm = 3.8                            % Resistance of coil (Ohms)
Km = 3.8                            % Force constant of shaker (N/A)
Kmv = Km/Rm                         % Force constant of shaker (N/V)
Lm = 0.5E-03                        % Inductance of the coil (Henry)

%% Define the state space system for the Mechanical system with force input

% The equation of motion for the mechanical system is given by
% $$ f = m\ddot{x} + b\dot{x} + kx $$
% We will now build a linear state space model of the plant. The mechanical state
% equations are given by
% $$ \mathbf{\dot{x}} = \mathbf{Ax} + \mathbf{Bu} $$
% $$ y = \mathbf{C}x + \mathbf{Du} $$
% where
% $$ \mathbf{x} = \left[ \begin{array} {c} x \\ \dot{x} \end{array} \right] $$
% $$ \mathbf{u} = f $$
% $$ \mathbf{A} = \left[ \begin{array} {cc} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{array} \right] $$
% $$ \mathbf{B} = \left[ \begin{array} {c} 0 \\ \frac{1}{m} \end{array} \right] $$
% $$ \mathbf{C} = \left[ \begin{array} {cc} -k & -b \end{array} \right] $$
% $$ \mathbf{D} = \left[ \begin{array} {c} 1 \end{array} \right] $$
A1 = [ 0   1; ... -k/m -b/m ];       % State matrix
B1 = [0 ; 1/m];                      % State input matrix
C1 = [-k -b];                        % State output matrix
D1 = 1;                              % Feedforward matrix

shaker_mechanical = ss(A1,B1,C1,D1); % Define the state space system
StateName = {'x(m)','\dot{x}(m/s)'}                             % Define the names of the states which are Displacement and Velocity
InputName = {'f_{coil}(N)'}                                     % Define the name(s) of the input(s) which is Coil force
OutputName = {'f_{reaction}(N)'}                               % Define the name(s) of the output(s) which is Reaction force
set(shaker_mechanical,'statename',StateName,'InputName',InputName,'OutputName',OutputName)

Question 1

The shaker system is an example of a harmonic oscillator. Calculate the poles of the open loop mechanical system and from this determine the system stability. Describe what this means in relation to the physical system.

%% Question 1

disp(' '); disp('1) The poles of the open loop Mechanical system are')
eig(A1)                     % From the state matrix directly, only the complex poles
pole(shaker_mechanical)     % From the state matrix directly, only the complex poles

Question 2

The pair of complex poles $$s = \omega_n \left( -\zeta_n \pm j\sqrt{1-\zeta_n^2} \right)$$ are typical of mechanical elements, where is the \(\omega_n\) natural frequency, \(\zeta_n\) is the damping ratio and \(\omega_n\sqrt{1-\zeta_n^2}\) is the resonance frequency. This is illustrated in Figure 4.
Determine the natural frequency of the mechanical spring-mass-damper system.

```matlab
%% Question 2
\% Information regarding complex poles, natural frequencies and damping ratios can be found at
\% <http://en.wikipedia.org/wiki/Damping_ratio>. Complex poles are given by
\% $$ s = -\omega_n \zeta_n \pm j \omega_n \sqrt{1-\zeta^2} $$
\% disp(' '); disp('2) The natural frequency (in rad/s) of the open loop Mechanical system is')
[Wn,Zeta] = damp(shaker_mechanical); \% Use the damp command to determine the poles of "shaker_mechanical"
Wn(1) \% Take only the first element as there are two complex poles
```

**Question 3**

Determine the damping ratio of the mechanical spring-mass-damper system.

```matlab
%% Question 3
\% disp(' '); disp('3) The damping ratio of the open loop Mechanical system is')
Zeta(1) \% Take only the first element as there are two complex poles
```

**Question 4**

Determine the resonance frequency (damped natural frequency) of the mechanical spring-mass-damper system.

```matlab
%% Question 4
\% disp(' '); disp('4) The resonance frequency (damped natural frequency) (in rad/s) of the open loop Mechanical system is')
temp = imag(pole(shaker_mechanical)); \% The resonance frequency is given by the imaginary part of the pole
temp(1) \% Take only the first element as there are two complex poles
```

**Question 5**

The locations of the poles will determine the frequency response function. Plot the frequency response on a Bode diagram. Look at the magnitude and frequency at the peak (damping controlled region) and relate this to the resonance frequency and quality factor defined as \( Q = \frac{1}{2\zeta} \). In this case, the quality factor indicates the gain increase when a system is driven at resonance compared to at high frequencies in the mass controlled region (when only the effect of the mass is felt).

```matlab
%% Question 5
\% disp(' '); disp('5) Look at the frequency response function of the system.;')
disp('Relate the peak in the response to the resonance frequency and damping ratio')
figure \% Open a new figure
(shaker_mechanical) \% Generate the Bode plot of "shaker_mechanical" using the "bode" command
grid \% Add a grid to the plot
```
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Figure 5: Schematic of the shaker system frequency response function

**Question 6**
The locations of the poles also determine the behaviour of the *Step response*. Plot the step response. Look at the period of the oscillations and relate this to the resonance frequency. Also determine the 2% *Settling time* (right click in graph Characteristics > Settling Time) and compare against the theoretical which is \( T_{2\%} = \frac{4}{\sigma} = \frac{4}{\omega_n \zeta_n} \).

```matlab
% Question 6
% The 2% settling time of a pole is given by
% $$ T_{2\%} = \frac{4}{\sigma} = \frac{4}{\omega_n \zeta_n} $$

disp('6) Look at the step response function of the system.')
disp('Relate the transient response to the resonance frequency and damping ratio')
figure                  % Open up a new figure
(shaker_mechanical) % Plot the step response of the "shaker_mechanical" system using the "Step" command.
grid                    % Add a grid to the plot
```
PART B

Electrical system differential equations

The differential equation describing the electrical subsystem for the shaker may be found using Kirchhoff’s law:

\[ L_m \ddot{i} + R_m i + K_m \dot{x} = V_{out} \quad (4) \]

which may be arranged in terms of the derivative of the current \( \dot{i} \)

\[ \dot{i} = \frac{V_{out} - R_m i - K_m \dot{x}}{L_m} \quad (5) \]

Now, the poles of the electrical subsystem may be obtained by taking the Laplace transform of Equations (4) (in the absence of motion or a voltage input), then factorising:

\[ i(s)(L_m s + R_m) \Rightarrow s = \frac{-R_m}{L_m} = 7600 \text{rad/s} \approx 1200 \text{Hz} \]

The force produced by the coil on the magnet is given by the product of the force constant and current,

\[ f = K_m i \quad (6) \]

Finally, the gain through the power amplifier has been adjusted to unity across the bandwidth of operation (10-200Hz). Therefore

\[ V_{in} = V_{out} = V \quad (7) \]

Full state equations for coupled mechanical and electrical system

We can now merge the simultaneous equations of the mechanical and electrical subsystems to give the fully coupled state equation:

\[
\begin{bmatrix}
\dot{x} \\
\dot{i}
\end{bmatrix}
= 
\begin{bmatrix}
0 & 1 \\
-\frac{k}{m} & -\frac{b}{m}
\end{bmatrix}
\begin{bmatrix}
x \\
\dot{x}
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
0
\end{bmatrix}
V
\]

(8)

Now the desired output of the system is the force induced by the motion of the mass, ie

\[ y = m \ddot{x} = \begin{bmatrix}
-k & -b & K_m
\end{bmatrix}
\begin{bmatrix}
x \\
\dot{x} \\
i
\end{bmatrix}
+ 0V
\]

(9)

Questions

Now build a state space model of the fully coupled equations using the code below.
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The coupled state equations are given by

\[
\dot{x} = Ax + Bu \\
y = f_{\text{reaction}} = m \ddot{x} = Cx + Du
\]

where

\[
A = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & \frac{K_m}{m} \\ 0 & -\frac{K_m}{L_m} & -\frac{R_m}{L_m} \end{bmatrix} \\
B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_m} \end{bmatrix} \\
C = \begin{bmatrix} -k & -b & K_m \end{bmatrix} \\
D = 0
\]

\[
A2 = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & \frac{K_m}{m} \\ 0 & -\frac{K_m}{L_m} & -\frac{R_m}{L_m} \end{bmatrix} \\
B2 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_m} \end{bmatrix} \\
C2 = \begin{bmatrix} -k & -b & K_m \end{bmatrix} \\
D2 = 0
\]

\[
\text{shaker\_coupled = ss( } ); \\
\text{InputName = \{'V_{coil}(V)\'} ; } \\
\text{OutputName = \{'f_{\text{reaction}}(N)\'} ; } \\
\text{StateName = \{'x(m)\', '\dot{x}(m/s)\', 'i(Amp)\'} ; } \\
\text{set(shaker\_coupled, 'statename', StateName, 'InputName', InputName, 'OutputName', OutputName); }
\]

Question 7

Calculate the open loop poles of the fully coupled plant.

\[
\text{shaker\_coupled = ss( } ); \\
\text{InputName = \{'V_{coil}(V)\'} ; } \\
\text{OutputName = \{'f_{\text{reaction}}(N)\'} ; } \\
\text{StateName = \{'x(m)\', '\dot{x}(m/s)\', 'i(Amp)\'} ; } \\
\text{set(shaker\_coupled, 'statename', StateName, 'InputName', InputName, 'OutputName', OutputName); }
\]

Question 8

\[
\text{co = ctrb(A2,B2)} \\
\text{co = ctrb(shaker\_coupled)} \\
\text{disp('The rank of the controllability matrix is')} \\
\text{rank(co)} \\
\text{disp('The condition number of the controllability matrix is')} \\
\text{cond(co)}
\]

Question 9

\[
\text{shaker\_mechanical = ss( } ); \\
\text{InputName = \{'V_{coil}(V)\'} ; } \\
\text{OutputName = \{'f_{\text{reaction}}(N)\'} ; } \\
\text{StateName = \{'x(m)\', '\dot{x}(m/s)\', 'i(Amp)\'} ; } \\
\text{set(shaker\_mechanical, 'statename', StateName, 'InputName', InputName, 'OutputName', OutputName); }
\]

Question 7

Calculate the open loop poles of the fully coupled plant.

\[
\text{shaker\_coupled = ss( } ); \\
\text{InputName = \{'V_{coil}(V)\'} ; } \\
\text{OutputName = \{'f_{\text{reaction}}(N)\'} ; } \\
\text{StateName = \{'x(m)\', '\dot{x}(m/s)\', 'i(Amp)\'} ; } \\
\text{set(shaker\_coupled, 'statename', StateName, 'InputName', InputName, 'OutputName', OutputName); }
\]

Question 8

\[
\text{co = ctrb(A2,B2)} \\
\text{co = ctrb(shaker\_coupled)} \\
\text{disp('The rank of the controllability matrix is')} \\
\text{rank(co)} \\
\text{disp('The condition number of the controllability matrix is')} \\
\text{cond(co)}
\]

Question 9

Calculate the controllability matrix of the mechanical system derived previously and determine if this format of the equations of motion is more controllable than the fully coupled model.
Reduced state equations for coupled mechanical and electrical system

Now since the inductance is extremely small, the state equations may become poorly conditioned when including the current term. Consequently, at the limit as the inductance approaches zero, the current becomes an algebraic function of the proof mass velocity and the supply voltage, i.e.

\[
R_m\dot{x} + K_m\dot{x} = V \quad \Rightarrow \quad i = -\frac{K_m}{R_m} \dot{x} + \frac{1}{R_m} V
\]  

(10)

Now the forces generated by the supply voltage is given by

\[
f = K_m i = K_m \left( -\frac{K_m}{R_m} \dot{x} + \frac{1}{R_m} V \right) = -\frac{K_m^2}{R_m} \dot{x} + \frac{K_m}{R_m} V
\]  

(11)

We can now merge Equations (2) and (11) to obtain the reduced order state equations of the coupled electro-mechanical system

\[
\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k_m}{m} & -\frac{b_m}{m} - \frac{K_m^2}{m R_m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \frac{K_m}{m R_m} V  
\]  

(12)

And the force generated by the proof mass is

\[
f = m\ddot{x} = m \left( \frac{-k}{m} \dot{x} - \frac{b}{m} \dot{x} - \frac{K_k}{m R_k} \right) = \frac{m K_m}{m R_m} V
\]  

(13)

Questions

Now build the state equations for the reduced order model using the code provided below.
Question 10
Calculate the controllability matrix of the reduced order system and determine if this format of the equations of motion is more controllable than the fully coupled model.

```matlab
% Question 10
disp(' '); disp('10) The controllability matrix of the reduced order system is')
Co = ctrb(     )                % Calculate the controllability matrix from the A and B matrix
Co = ctrb(              )       % Alternatively, calculate the controllability matrix from the "shaker_reduced" state space system
disp('The rank of the controllability matrix is')
rank(Co)                        % Calculate the rank of the Co matrix using the "rank" command
disp('The condition number of the controllability matrix is')
cond(Co)                        % Calculate the condition number of the Co matrix using the "cond" command
```

Question 11
Calculate the open loop poles of the reduced order system by hand by solving det(sI-A)=0.

Repeat the exercise in Matlab by calculating the poles first using the damp command, and then secondly solving det(sI-A)=0 symbolically.

```matlab
% Question 11
disp(' '); disp('11) Calculate the poles of the open loop reduced order system and compare against det(sI-A)=0')
damp(              )                % Indirectly from the (A matrix) in the "shaker_reduced" state space model
% Calculate the poles symbolically
syms s                              % Define a symbolic variable s
I = eye(2)                          % Define the 2x2 identity matrix
char_eqn = det(s*I-A3)              % Characteristic equation
poles_symb = solve(char_eqn)        % Roots (poles) of characteristic equation
double(poles_symb)                  % Convert rational polynomial to numeric form
```

Question 12
Plot the frequency response functions for all three plants on a Bode diagram. Note any similarities and differences seen. Relate the features on the plot to the locations of the poles.

```matlab
% Question 12
disp(' '); disp('12) Look at the frequency response functions of all three systems and compare')
figure                                                              % Open a new figure
bode(shaker_mechanical, shaker_coupled, shaker_reduced)             % Generate a Bode plot of all three plants using the "bode" command
grid                                                                % Add a grid
('Mechanical','Coupled','Reduced Order','Location','Best')    % Add a legend using the "legend" command
```
PART C

Control System Design

For this part we are only going to consider the reduced order model.

**Question 13**

Convert the state equations in to the Laplace domain by solving the following by hand: \( C \times \text{inv}(sI-A) \times B + D \). Repeat the exercise in Matlab converting the state space system into the Laplace domain both as a rational polynomial and zero-pole-gain form. Also solve \( C \times \text{inv}(sI-A) \times B + D \) symbolically.

**Question 14**

Calculate the zeros of the transfer function by solving \( \text{det}(sI-A, B; C, D)=0 \) by hand and compare against the transfer function derived in Question 13. Repeat the exercise in Matlab using the \texttt{zero} command and also by solving \( \text{det}(sI-A, B; C, D)=0 \) symbolically.

**Question 15**

Observability, like controllability, is another important concept in control, and it typically denotes the ability to infer the internal states of a system from measurements made by a set of external sensors. Calculate the observability matrix of the reduced order plant and determine if the plant is observable.
Question 16
You will now design a full state feedback controller via pole placement to move the two lightly damped poles from 46 Hz to 10 Hz and heavily damped (with a damping ratio of sqrt(1/2)).

```matlab
% disp('16) Design a controller using pole placement for poles with natural frequency 10Hz and damping ratio sqrt(1/2)')
P = (10*2*pi)*[exp(i*3*pi/4) exp(-i*3*pi/4)] % Desired closed loop poles (rad/s)
K = place(shaker_reduced.A,shaker_reduced.B,P) % Control gains via a robust pole placement process using the 'place' command
K = place(shaker_reduced.A,shaker_reduced.B,P) % Control gains via Ackermann's formula using the 'acker' command
```

Question 17
Define the state space system of the closed loop and determine the locations of the closed loop poles.

```matlab
% disp('17) Define the closed loop state space system and determine the locations of closed loop poles')
% Define the closed loop state space system
shaker_fsf = ss(A3-B3*K, B3, C3-D3*K, D3); % Note the modified state and output matrices.
set(shaker_fsf,'statename',StateName,'InputName',InputName,'OutputName',OutputName)
shaker_fsf
% Closed loop poles are given by det(sI-A-BK)=0
damp(shaker_fsf) % Poles from the 'shaker_fsf' state space system using the 'damp' command
P = eig(A3-B3*K) % Alternatively, these may be obtained directly from the eigenvalues of the closed loop 'A' matrix using 'eig'
```

Question 18
Plot step response and frequency response of both the open and closed loop transfer functions and compare. Compare the features against the locations of the poles.

```matlab
% disp('18) Look at the response of the controlled plant')
% Now look at the step response
figure % Open a new figure
(shaker_reduced, shaker_fsf) % Plot the step response of the open and closed loop plants
grid % Add a grid
legend('Open loop Reduced','Closed loop Reduced FSF','Location','Best') % Add a legend to the plot
% Look at the Bode plot
figure % Open a new figure
(grid, shaker_fsf) % Generate the Bode plot of the open and closed loop plants
grid % Add a grid
legend('Open loop Reduced','Closed loop Reduced FSF','Location','Best') % Add a legend to the plot
```

Question 19
Derive the three separate output matrices to sense only the displacement of the proof mass, only the velocity of the proof mass or only the control voltage into the coil.

```matlab
% disp('19) The state output matrix and direct transmission matrix to measure displacement is')
C = [1 0]
D = 0
% disp('19) The state output matrix and direct transmission matrix to measure velocity is')
C = [0 1]
D = 0
% disp('19) The state output matrix and direct transmission matrix to measure the control voltage is')
C = -K
D = 0
```
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**Question 20**

It is possible to increase the number of sensors in a state space model by adding more rows to the state output equation (C & D matrices). Augment the model so that it is possible to sense the reaction force, the displacement and velocity of the proof mass, and the control voltage to the coil.

\[
\begin{align*}
\text{C3} &= \begin{bmatrix} C3-D3*K; 1 & 0 & 0 & K \end{bmatrix} & \text{Augmented C matrix to see closed loop reaction force, displacement and velocity of proof mass, and control voltage} \\
\text{D3} &= \begin{bmatrix} D3; 0; 0; 0 \end{bmatrix} & \text{Augmented D matrix to see closed loop reaction force, displacement and velocity of proof mass, and control voltage}
\end{align*}
\]

\[
\text{shaker_4outputs} = \text{ss}(A3-B3*K, B3, C3, D3);
\]

\[
\text{OutputName} = \{'f_{reaction}(N)', 'x(m)', '\dot{x}(m/s)', 'V_{control}(V)\'}
\]

\[
\text{set(shaker_4outputs,'statename',StateName,'InputName',InputName,'OutputName',OutputName)}
\]

**Question 21**

Now plot the step response of the closed loop system, showing all four outputs. Determine the peak force, the steady state force, steady state displacement and steady state velocity, and the steady state control voltage arising from the application of a constant unit voltage into the coils.

\[
\begin{align*}
\text{figure} &\quad \text{Open a new figure} \\
\text{shaker_4outputs} &\quad \text{Plot the response of all four outputs to a unit voltage in to the coil} \\
\text{grid} &\quad \text{Plot a grid}
\end{align*}
\]