Parameters for optimising the forces between linear multipole magnet arrays

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***Abstract*—Multipole magnet arrays have the potential to achieve greater forces than homogeneous magnets for linear spring applications. This paper investigates the effects of varying key parameters of linear multipole magnet arrays in relation to their force bearing potential. Equal sized arrays in repulsive configurations are vertically displaced to each other; only vertical forces are compared. The force versus displacement characteristic is dependent on the aspect ratio of the arrays, the wavelength of magnetisation, and the total number of magnets used in the array. Some general design guidelines for optimising the repulsive forces are established based on the results.**

I. Introduction

Linear springs constructed with permanent magnets can behave in interesting ways. In levitation contexts, the natural gravity-opposing characteristic with zero power input makes permanent magnets an appealing choice for force generation (e.g., see Íñiguez and Raposo 2009). In the context of supporting a variable-mass load for vibration isolation, the non-linear force vs. displacement characteristic decreases the amount of variability in the resonance frequency of the structure, since as the mass increases and closes the gap between the magnets, the stiffness also increases (Bonisoli and Vigliani 2007). When used in attraction, the negative stiffness can be used to decrease the resonance frequencies of a supported mass, applicable for 'high-static–low-dynamic' or 'quasi–zero stiffness' springs (Carrella, Brennan, Waters, and Shin 2008; Robertson, Kidner, Cazzolato, and Zander 2009).

With well-known closed form solutions for calculating the forces between cuboid permanent magnets of parallel magnetisation (Akoun and Yonnet 1984) and solutions for the forces with orthogonal magnetisations recently published (Janssen et al. 2009; Allag, Yonnet, Fassenet, and Latreche 2009), it is now possible to analyse a wide variety of magnet configurations that previously required semi-analytical or finite element analsis techniques. This paper investigates the force characteristics between linear multipole magnet arrays as a function of array size and magnet arrangement using cuboid-shaped magnets. The forces are calculated using the force equations derived by the above researchers.

The results presented in this paper are reproducible (Buckheit and Donoho 1995) with code located at http://www.github.com/wspr/magcode. This is a Matlab software package written by the authors for calculating the forces between magnets and multipole arrays of magnets, and is freely available to be used by the public. The directory `examples/magspring/` contains the code that has been used to directly generate the figures in this paper. Each figure caption refers to the file from which it is generated.

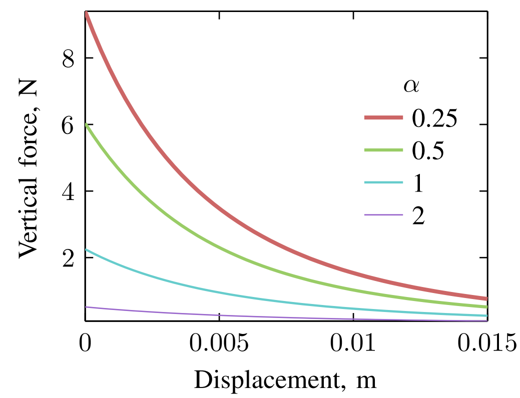


Fig. 1: Force vs. displacement for pairs of repulsive magnets of constant volume each of $(10\,\mathrm{mm})^3$, comparing various magnet height-to-length ratios α. Calculated in the file 'mag_ratio.m'.

II. Forces between single magnets

Consider two equal cuboid-shaped permanent magnets facing in repulsion with vertical magnetisation and only vertical displacement between them. The shape of the cuboid has a dramatic affect on the force vs. displacement characteristic of the system. Assume square facing sides and a height-to-length ratio of α. For fixed magnet volume V, the height of the cuboid is $\sqrt[3]{V\alpha^2}$ and the face size length is $\sqrt[3]{V/\alpha}$. The force vs. displacement characteristic between two such magnets each with constant volume $V = (10\,\mathrm{mm})^3$ over a range of aspect ratios α is shown in Figure 1. Displacement is measured between the near faces of the magnets. (Note that all forces in this paper are calculated assuming a magnetisation of $1\,\mathrm{T}$ for all magnets. This value has been chosen to in effect normalise the output forces by the magnetisation strength.) These results show that for a constant volume of magnetic material a greater facing area yields greater forces.

III. Linear multipole magnet arrays

Large, flat, thin magnets can be difficult to obtain and hard to work with. While multiple smaller magnets can be stacked together to approximate a single large magnet, greater forces can be achieved by varying the magnetisation pattern between adjacent magnets in the stack. In the late '70s it was discovered that magnets with sinusoidal magnetisation

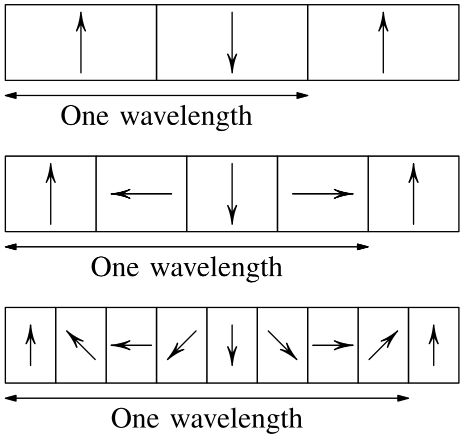


Fig. 2: Three Halbach arrays of equal length, facing up, each with a single wavelength of magnetisation and composed of $R \in \{2, 4, 8\}$ magnets per wavelength respectively.

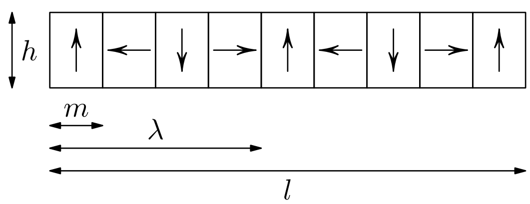


Fig. 3: Geometry of a linear Halbach array with four magnets of length m per wavelength of magnetisation λ. This array contains two wavelengths of magnetisation with an end magnet for symmetry, i.e., $W = 2$ and $R = 4$.

produced single-sided magnetic fields (Halbach 1980; Shute, Mallinson, Wilton, and Mapps 2000), although it was known much earlier for magnetic bearings that stacks of shorter ring magnets with alternating magnetisations produced greater forces than a longer single ring magnet (Backers 1961).

IV. Geometry and magnets arrangement

In this paper we will consider a linear stack of magnets with N magnets aligned along an horizontal axis (often referred to as a linear Halbach array); planar and volumetric stacks that have multiple magnets in the other directions will not be considered here. As the number of magnets per wavelength of magnetisation R increases, the magnetisation pattern of the array more closely approximates true sinusoidal magnetisation, as shown in Figure 2.

The magnetic flux pattern of the array is dependent also on the wavelength of magnetisation λ. As the wavelength of magnetisation decreases, the total number of magnets used in the array increases, for a fixed array length l. The relationship between wavelength, array length, and number of magnets is shown in Figure 3. Note the one extra magnet included in Figures 2 and 3 such that the total number of wavelengths $W = [l - m]/\lambda$ and the total number of magnets $N = WR + 1$. This magnet is necessary to balance the forces in the horizontal direction such that only vertical forces are generated between the arrays.

There are two independent variables to consider when choosing the parameters for a linear Halbach array of a certain size: number of magnets per wavelength R, and total number of wavelengths W in the array.

A. *Varying magnetisation discretisation and wavelength*

Consider two linear Halbach arrays of equal size with height $h = 10\,\mathrm{mm}$, square cross section, and length $l = 100\,\mathrm{mm}$. Their strong sides are aligned towards each other and their magnetisation pattern is such that there is a repulsive force between them. The vertical displacement between their centres is δ which can be normalised by the height of the arrays; $\delta/h = 1$ corresponds to the position at which the faces of the two arrays are touching. By calculating the forces between the arrays using superposition of the forces between each permutation of magnet pairs in the two arrays (Allag, Yonnet, and Latreche 2009), the force vs. normalised vertical displacement was calculated for the number of magnets per wavelength $R \in \{2, 4, 8\}$ and the number of wavelengths $W \in \{1, 2, 4\}$ and compared to the forces generated between a pair of equivalently-sized magnets of homogeneous magnetisation. These results are shown in Figure 4.

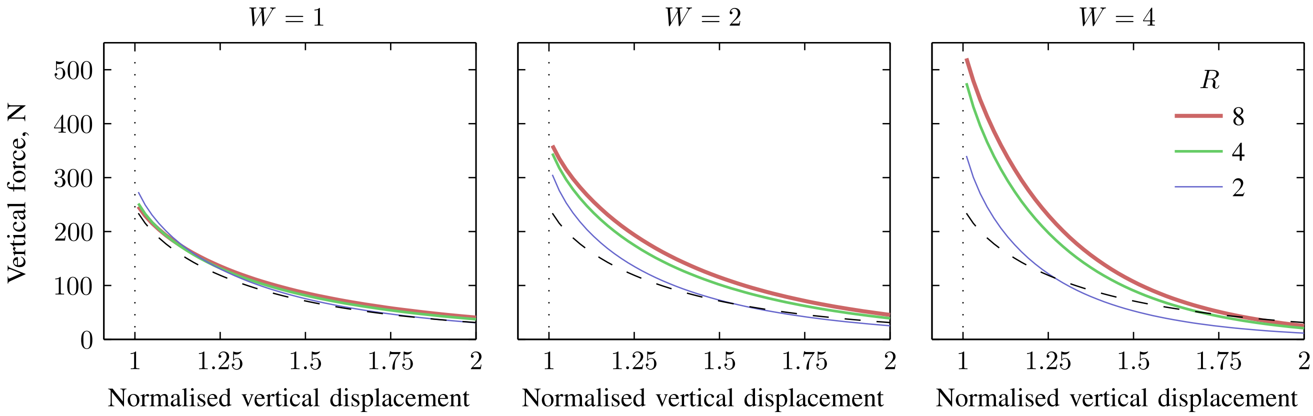
It can be seen in Figure 4(a) that for a small number of wavelengths, the discretisation of the magnetisation makes little difference to the force characteristic. But as the number of wavelengths increases the number of magnets per wavelength has an increasing effect. As seen most prominently in Figure 4(c), increasing the number of magnets per wavelength R increases the forces over all values of displacement considered.

Therefore, as a general design guideline, it is only necessary to use a large number of magnets per wavelength if there is at least several wavelengths of magnetisation in total in the array. In the results shown in Figure 4, the ratio in forces between $R = 2$ and $R = 4$ is greater than the ratio in forces between $R = 4$ and $R = 8$; most of the benefit of increasing the number of magnets is realised using four magnets per wavelength of magnetisation (i.e., 90° rotations between successive magnets such as shown in Figure 3). In cases where there are many more wavelengths of magnetisation again (as shown in Figure 5 later), there is a greater advantage to using $R = 8$ over $R = 4$. Therefore, the greater number of wavelengths of magnetisation, the greater the force improvement in increasing the number of magnets per wavelength.

The use of multipole arrays can have a significant effect on the useful range of the force/displacement characteristic. As the number of wavelengths increases, the magnetic field of each array becomes stronger but the magnetic field lines exhibit smaller excursions outside the magnet array before returning. Thus, the forces become stronger but over a smaller displacement, and therefore the stiffness of the magnetic spring is increased as well. For some purposes and in some cases, this can be detrimental in that it can increase the resonance frequency of the system, resulting in poorer vibration isolation properties.

B. *Constant number of magnets*

The results shown previously have in general indicated that improvements to the force characteristic are seen with



(a) (b) (c)

Fig. 4: Force vs. displacement normalised by the array height h between two facing linear Halbach arrays with a varying number of magnets per wavelength R and a varying number of wavelengths of magnetisation W. The dashed line is the force between two single magnets of equal size to the arrays. Calculated in the file 'multipole_compare.m'.

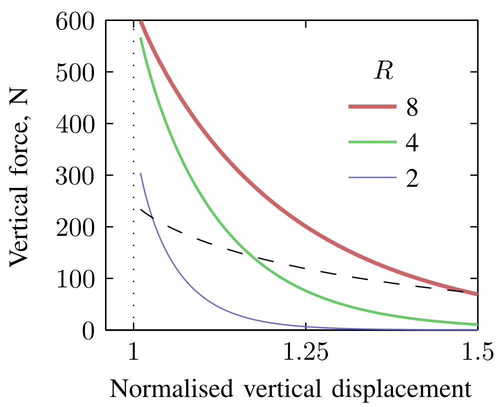


Fig. 5: Force characteristic with arrays each composed of fifty magnets of length $m = 2\,\mathrm{mm}$, over a variety of number of magnets per wavelength R. The dashed line is the force between two homogeneous magnets of length $100\,\mathrm{mm}$. Calculated in 'multipole_const_Nmag.m'.

a greater number of magnets. However, given a minimum magnet thickness that can be fabricated, and hence for a given array length a maximum number of magnets in total, the question arises: is it better to maximise the number of wavelengths W or the number of magnets per wavelength R? Consider an array of the same outer dimensions as the previous example composed of magnets each of length $m = 2\,\mathrm{mm}$ and of cross-sectional area $10\,\mathrm{mm} \times 10\,\mathrm{mm}$, such that there are 50 magnets in the array. The force characteristic for this system, again with $R \in \{2, 4, 8\}$, is shown in Figure 5. In this extreme example with large W, the strong region of the field is close to the surfaces of the arrays and there is considerable difference in the curves for each value of R; maximising R produces stronger results providing there are sufficiently many wavelengths of magnetisation along the length of the array. When W is small (say $W < 5$) for a fixed magnet size, these general results do not hold and the design possibilities must be evaluated on a case-by-case basis.

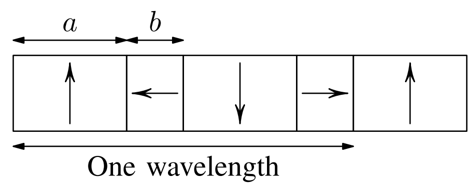


Fig. 6: Schematic of a four-magnet Halbach array with variable magnet sizes. The number of magnets per wavelength $R = 4$ for all arrays of this type unless $b = 0$, in which case $R = 2$.

C. Non-equal magnet sizes

While the force characteristic of an eight-magnet wavelength ($R = 8$) array can outperform the four-magnet ($R = 4$) array, the latter can be improved in some cases by adjusting the relative sizes of the magnets in the array. Consider the four-magnet array shown in Figure 6 in which the horizontally-polarised magnets of length b are smaller than the vertically-polarised magnets of length a. Magnet size ratio $\gamma = b/a$ is the measure used here to compare different array configurations, for which $\gamma = 0$ corresponds to an array composed only of vertically-oriented magnets, and $\gamma = 1$ corresponds to equally-sized magnets of both horizontal and vertical magnetisations (as considered previously in this paper).

Figure 7 compares the force characteristic with a variety of magnet size ratios for arrays composed of nine magnets (i.e., two wavelengths of magnetisation with a symmetry magnet), of length $l = 100\,\mathrm{mm}$, and of cross-sectional area $10\,\mathrm{mm} \times 10\,\mathrm{mm}$. As expected from the previous results, $\gamma = 0$ results in smaller forces than for $\gamma = 1$; however, $\gamma = 0.5$ results in slightly greater forces again: an increase of 5% at a displacement of approximately $\delta = 1.3h$, tapering off as the displacement increases. (This value of γ is close to optimum for this system; see Figure 8.) This result can be justified intuitively with the recognition that there is a stronger vertical force between opposing vertically-polarised magnets than between horizontally-polarised magnets; dedi-

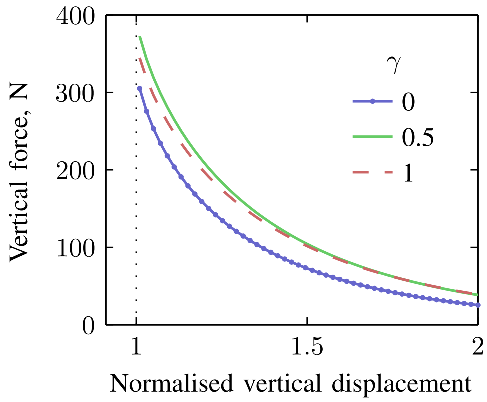


Fig. 7: Force characteristic between two modified Halbach arrays of $W = 2$ with magnet length ratio γ between the sizes of vertically- to horizontally-polarised magnets. Calculated in 'linear_quasi_example.m'.

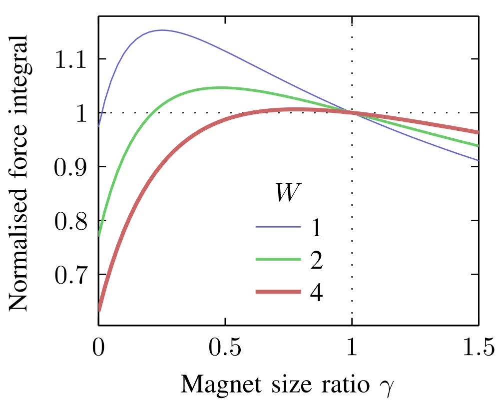


Fig. 8: Integral of the force–displacement characteristic vs. magnet length ratio γ of two modified Halbach arrays of size $100\,\mathrm{mm} \times 10\,\mathrm{mm} \times 10\,\mathrm{mm}$ over a displacement range of $10\,\mathrm{mm}$ to $20\,\mathrm{mm}$, shown with varying number of wavelengths of magnetisation W. Calculated in 'linear_quasi_ratios.m'.

cating a greater proportion of the magnet volume to the vertical magnets yields an increase in the total force.

However, as the number of wavelengths of magnetisation W increases, there is a decrease in the improvement offered by reducing the size of the horizontal magnets. This can be quantified by comparing the integral of force over the displacement range of interest for a variety of magnet length ratios γ. In Figure 8, such results are shown (for the same arrays discussed previously) comparing the relative difference of the force–displacement integral as a function of the magnet length ratio, normalised by the integral results for $\gamma = 1$. Since the force improvement with adjusting γ is only significant for low numbers of wavelength of magnetisation, this technique is only suitable for increasing the forces when a small total number of magnets are to be used, perhaps for ease of construction of the magnet array. Otherwise, it is more efficient simply to increase the number of magnets than to change the magnet size ratio.

V. Conclusion

In conclusion, in optimising the forces between linear Halbach magnet arrays, it has been shown that there is a relationship on the force vs. displacement characteristic from both the wavelength of magnetisation and the number of magnets in the array. As the wavelength of magnetisation decreases while keeping the array length constant, the effect of increasing the number of magnets per wavelength increases. In order to achieve significantly larger forces over homogeneous magnetisation, a large number of magnets should be used; a minimum of around $16 + 1$ magnets with 90° magnetisation rotations or $32 + 1$ magnets with 45° magnetisation rotations. When only a small number of magnets are used, small improvements to the forces can be achieved by using magnets of smaller volume which are magnetised parallel to the array length and magnets of larger volume which are polarised in the facing direction.

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