

# Maximising the force between two cuboid magnets

Will Robertson, Ben Cazzolato, and Anthony Zander  
will.robertson@adelaide.edu.au

School of Mechanical Engineering, University of Adelaide, SA, Australia

**Abstract**—Recently, we wrote that to maximise the force between two magnets of fixed volume, the magnet dimensions should be chosen to be as thin as possible in the direction of magnetisation. This was incorrect, and we would like to clarify this point.

In a recent paper (Robertson, Cazzolato, and Zander 2010), we wrote that ‘for a constant volume of magnetic material, a greater facing area yields greater forces’; unfortunately, Figure 1 in that publication was incorrect, and this led us to an invalid conclusion on this point.

Consider the basic magnetic spring shown in Figure 1, consisting of two magnets separated by a displacement,  $x$  (measured between the near faces), and generating a repulsive force,  $F$ , between them. The magnets have square facing sides, a height-to-width ratio of  $\gamma = a/b$ , and a fixed volume  $V$ ; the height of each magnet is  $a = \sqrt[3]{V\gamma^2}$  and the face size width (and length into the page) is  $b = \sqrt[3]{V/\gamma}$ .

The magnetic forces between the magnets can be calculated by applying the theory of Akoun and Yonnet (1984), where the force  $F = F_m(V, \gamma, x)$  is a function of magnet volume  $V$ , size ratio  $\gamma$ , and displacement  $x$ .

Such forces were calculated between these magnets for a magnet volume  $V = (10 \text{ mm})^3$  over a displacement  $x$  from 0 mm to 10 mm and a magnet size ratio  $\gamma$  from 0.1 to 1. Note that the forces were calculated with a magnetisation of 1 T for both magnets, essentially normalising the output forces by the magnetisation strength. The MATLAB code used to calculate these results is located in the script `examples/mag_ratio.m` in the code repository <http://github.com/wspr/magcode>.

In order to compare the force versus displacement characteristics for a range of magnet size ratios, we will normalise the forces by the force  $F_s = F_m(V, 1, x)$ ; that is, the force for a magnet size ratio  $\gamma = 1$ . Figures 2 and 3 show the normalised force  $\bar{F} = F/F_s$  as a function of displacement  $x$

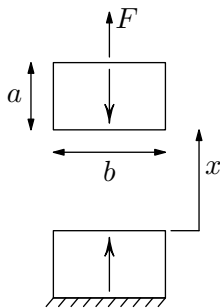


Fig. 1: Schematic of the magnetic spring. The magnets have square facing sides and extend a distance of  $b$  in the direction toward the reader. When  $x = 0$  the magnet faces are touching.

over a range of magnet size ratios  $\gamma$ . The figures are drawn as separate graphs in order to avoid overlap of the curves; size ratio  $\gamma$  varies from 0 to 0.4 in Figure 2 and from 0.4 to 0.8 in Figure 3. It can be seen from the two graphs that a magnet size ratio  $\gamma$  of around 0.4 produces the greatest forces; for values both less and greater than 0.4, the normalised force curves decrease.

Some small overlap in the force curves for  $\gamma = 0.4$  and  $\gamma = 0.5$  is seen in Figure 3. This indicates that the optimum magnet size ratio (to maximise the force) is dependent on the displacement between the magnets. Figure 4 shows the magnet force varying as a function of magnet size ratio  $\gamma$  with a set of curves corresponding to fixed displacements from 1 mm to 10 mm. For each curve, there is a local maxima in the force; this corresponds to the magnet size ratio that produces the greatest force at that displacement. While the magnet size ratio that produces the greatest forces varies with displacement, the graph shows that the optimum magnet size ratio remains around  $\gamma \approx 0.4$ .

Therefore, the statement in our previous paper is incorrect and the magnet thickness should not be made as thin as possible; to maximise the force for a fixed magnet volume, the thickness of the magnets should be minimised to not less than 40% of their width.

While the main conclusions of the original publication are not invalidated by this correction, there is now scope for further analysis of the force behaviour between multipole arrays of varying height-to-width (and perhaps height-to-wavelength) aspect ratios.

## REFERENCES

- Akoun, Gilles and Jean-Paul Yonnet (Sept. 1984). “3D analytical calculation of the forces exerted between two cuboidal magnets”. In: *IEEE Transactions on Magnetics* MAG-20.5, pp. 1962–1964. DOI: 10.1109/TMAG.1984.1063554.
- Robertson, W., B. Cazzolato, and A. Zander (Dec. 2010). “Parameters for Optimizing the Forces Between Linear Multipole Magnet Arrays”. In: *Magnetics Letters, IEEE* 1. DOI: 10.1109/LMAG.2010.2047716.

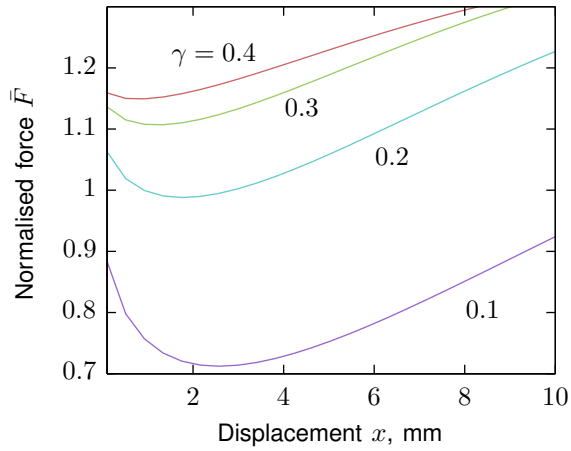


Fig. 2: Normalised force  $\bar{F} = F/F_s$  as a function of displacement  $x$  for a set of magnet ratios  $\gamma = \{0.1, 0.2, 0.3, 0.4\}$ .

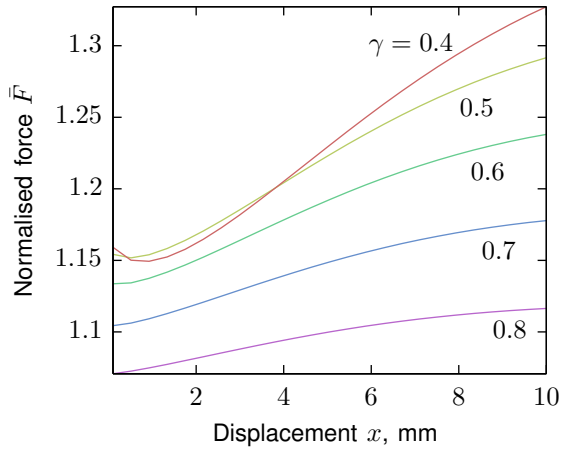


Fig. 3: Normalised force  $\bar{F} = F/F_s$  as a function of displacement  $x$  for a set of magnet ratios  $\gamma = \{0.4, 0.5, 0.6, 0.7, 0.8\}$ .

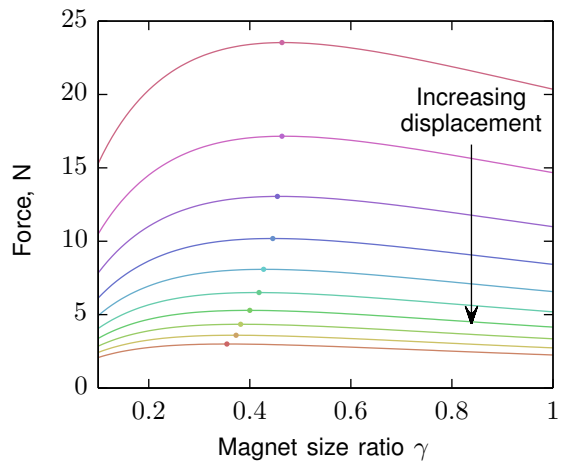


Fig. 4: Force between two magnets as a function of magnet height–width ratio  $\gamma = a/b$  at a number of fixed displacements from 1 mm to 10 mm. The positions of maximum force are shown with dots, indicating that magnet size ratio to achieve the maximum force is in the region around  $\gamma = 0.4$ .