Nonlinear control of a zero stiffness magnetic spring

Will Robertson
A/Profs Ben Cazzolato & Anthony Zander

July 12, 2007

Here I am

What shall I talk about?

Broadly speaking, two points:

1. Zero stiffness (and vibration isolation)
2. Nonlinear control

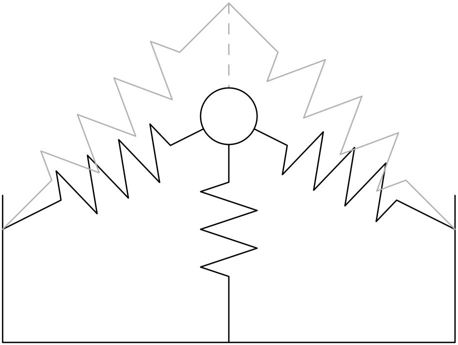
What you say 'zero stiffness'?

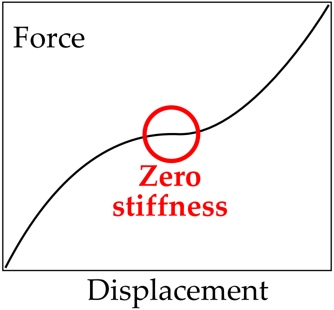
Well, quasi–*zero stiffness*

1. Russian guys (1989)
   'Vibration protecting and measuring systems with quasi-zero stiffness'
2. Gert-Jan Nijsse (Twente, 2001)
3. Alex Carrella (ISVR, now)
4. Me (Adelaide, now)

Zero stiffness mechanisms

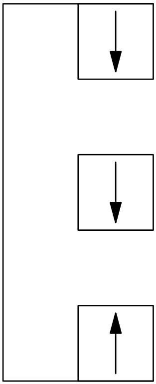
Mechanical springs

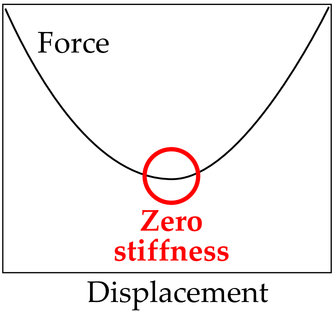




Zero stiffness mechanisms

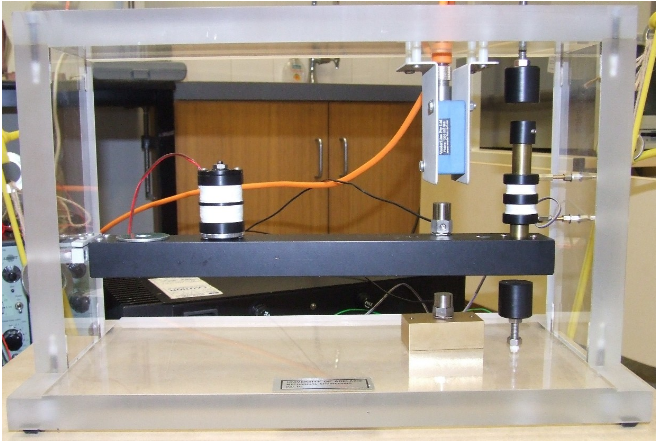
Unstable magnets





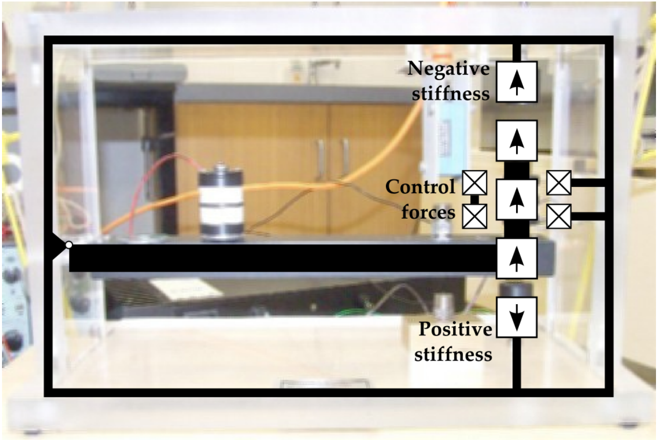
Zero stiffness mechanisms

Experimental rig

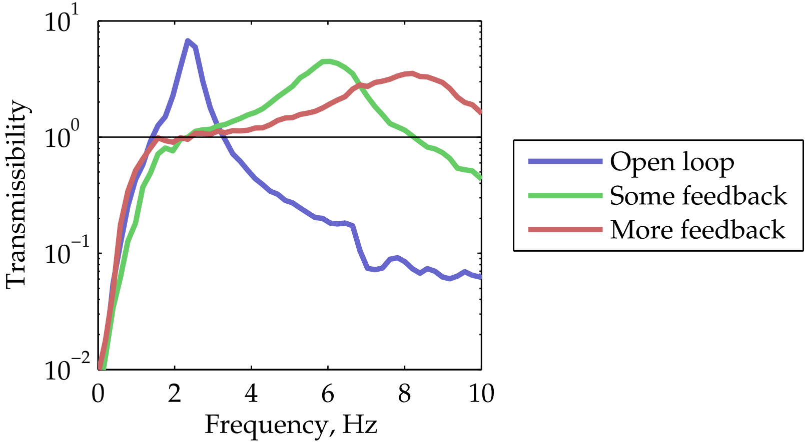


Zero stiffness mechanisms

Experimental rig



Rig characteristics



Nonlinear system model

A coupled mechanical-electrical system:

$$F_{\text{total}} = F_{\text{gravity}} + F_{\text{magnets}} + F_{\text{coil}} + F_{\text{damping}}$$
$$V_{\text{total}} = V_{\text{input}} + V_{\text{resistance}} + V_{\text{back-emf}}$$

That is:

$$M\ddot{x} = -Mg + F_{m0} + K_m(x - x_0)^2 + K_c I - C_d \dot{x}$$
$$L\dot{I} = Gu - IR - K_c \dot{x}$$

Nonlinear system model

A coupled mechanical-electrical system:

$$F_{\text{total}} = F_{\text{gravity}} + F_{\text{magnets}} + F_{\text{coil}} + F_{\text{damping}}$$
$$V_{\text{total}} = V_{\text{input}} + V_{\text{resistance}} + V_{\text{back-emf}}$$

That is:

$$M\ddot{x} = -Mg + F_{mo} + K_m(x - x_o)^2 + K_c I - C_d\dot{x}$$
$$L\dot{I} = Gu - IR - K_c\dot{x}$$

Nonlinear system model

A coupled mechanical-electrical system:

$$F_{\text{total}} = F_{\text{gravity}} + F_{\text{magnets}} + F_{\text{coil}} + F_{\text{damping}}$$
$$V_{\text{total}} = V_{\text{input}} + V_{\text{resistance}} + V_{\text{back-emf}}$$

That is:

$$M\ddot{x} = -Mg + F_{m0} + K_m(x - x_0)^2 + K_c I - C_d\dot{x}$$
$$L\dot{I} = Gu - IR - K_c\dot{x}$$

Nonlinear control

Krstic is my hero

Backstepping controllers for 'lower triangular' systems such as: (albeit any number of states are possible)

$$\begin{aligned} \dot{x}_1 &= x_2 + \varphi_1(x_1)^{\mathrm{T}}\theta, \\ \dot{x}_2 &= x_3 + \varphi_2(x_1, x_2)^{\mathrm{T}}\theta, \\ \dot{x}_3 &= u + \varphi_3(x_1, x_2, x_3)^{\mathrm{T}}\theta, \end{aligned}$$

with

- nonlinear φ_i known
- constant θ unknown.

And there are classes of controllers for such systems. . .

Nonlinear control

Backstepping with tuning functions

$$z_1 = x_1 - y, \quad z_2 = x_2 - \alpha_1$$
$$z_3 = x_3 - \alpha_2, \quad u = \varrho_3\alpha_3$$

$$\alpha_1 = -c_1 z_1$$

$$\begin{aligned}\alpha_3 = &-\kappa_3 z_3 x_2^2 - \vartheta_5 x_2 + \partial_{x_1}\{\alpha_2\}x_2 \\ &-(x_1^4 + x_1^2 \\ &+ x_2^2 + x_3^2 + 1)\kappa_3 z_3 \partial_{x_2}\{\alpha_2\}^2 \\ &+ \partial_\vartheta\{\alpha_3\}\dot{\vartheta} - x_3\vartheta_6 - \beta_2 z_2 \\ &-\kappa_3 z_2^2 z_3 - c_3 z_3 - x_3^2\kappa_3 z_3 \\ &+ (\vartheta_1 + x_1(\vartheta_2 + x_1\vartheta_3) \\ &+ x_2\vartheta_4 + x_3(\beta_2 + 2\kappa_3 z_2 z_3))\partial_{x_2}\{\alpha_2\} \\ &+ \dot{\varrho}_2\partial_{\varrho_2}\{\alpha_2\}\end{aligned}$$

$$\begin{aligned}\alpha_2 = \varrho_2(&-\vartheta_3 x_1^2 - \vartheta_2 x_1 - x_1 \\ &+ y - \vartheta_1 - x_2\vartheta_4 - c_2 z_2 \\ &-(x_1^4 + x_1^2 + x_2^2 + 1)\kappa_2 z_2 \\ &+ x_2\partial_{x_1}\{\alpha_1\})\end{aligned}$$

$$\begin{aligned}\dot{\vartheta} = \Gamma[&z_2 - z_3\partial_{x_2}\{\alpha_2\}, \\ &x_1(z_2 - z_3\partial_{x_2}\{\alpha_2\}), \\ &x_1^2(z_2 - z_3\partial_{x_2}\{\alpha_2\}), \\ &x_2(z_2 - z_3\partial_{x_2}\{\alpha_2\}), \\ &x_2 z_3, x_3 z_3]^{\mathrm{T}},\end{aligned}$$

$$\dot{\beta}_2 = -\gamma_2 z_3(-x_2 + \alpha_1 + x_3\partial_{x_2}\{\alpha_2\}),$$
$$\dot{\varrho}_2 = -\gamma_2 z_2\alpha_2/\varrho_2, \quad \dot{\varrho}_3 = -\gamma_3 z_3\alpha_3,$$

Nonlinear control

Backstepping with tuning functions

$$z_1 = x_1 - y, \quad z_2 = x_2 - \alpha_1$$
$$z_3 = x_3 - \alpha_2, \quad u = \varrho_3 \alpha_3$$

$$\alpha_1 = -c_1 z_1$$

$$\begin{aligned}\alpha_3 = &-\kappa_3 z_3 x_2^2 - \vartheta_5 x_2 + \partial_{x_1}\{\alpha_2\} x_2 \\ &- (x_1^4 + x_1^2 \\ &+ x_2^2 + x_3^2 + 1)\kappa_3 z_3 \partial_{x_2}\{\alpha_2\}^2 \\ &+ \partial_{\vartheta}\{\alpha_3\}\dot{\vartheta} - x_3 \vartheta_6 - \beta_2 z_2 \\ &- \kappa_3 z_2^2 z_3 - c_3 z_3 - x_3^2 \kappa_3 z_3 \\ &+ (\vartheta_1 + x_1(\vartheta_2 + x_1 \vartheta_3) \\ &+ x_2 \vartheta_4 + x_3(\beta_2 + 2\kappa_3 z_2 z_3))\partial_{x_2}\{\alpha_2\} \\ &+ \dot{\varrho}_2 \partial_{\varrho_2}\{\alpha_2\}\end{aligned}$$

$$\begin{aligned}\alpha_2 = \varrho_2(&-\vartheta_3 x_1^2 - \vartheta_2 x_1 - x_1 \\ &+ y - \vartheta_1 - x_2 \vartheta_4 - c_2 z_2 \\ &- (x_1^4 + x_1^2 + x_2^2 + 1)\kappa_2 z_2 \\ &+ x_2 \partial_{x_1}\{\alpha_1\})\end{aligned}$$

$$\begin{aligned}\dot{\vartheta} = \Gamma[&z_2 - z_3 \partial_{x_2}\{\alpha_2\}, \\ &x_1(z_2 - z_3 \partial_{x_2}\{\alpha_2\}), \\ &x_1^2(z_2 - z_3 \partial_{x_2}\{\alpha_2\}), \\ &x_2(z_2 - z_3 \partial_{x_2}\{\alpha_2\}), \\ &x_2 z_3, x_3 z_3]^{\mathrm{T}},\end{aligned}$$

$$\dot{\beta}_2 = -\gamma_2 z_3(-x_2 + \alpha_1 + x_3 \partial_{x_2}\{\alpha_2\}),$$
$$\dot{\varrho}_2 = -\gamma_2 z_2 \alpha_2 / \varrho_2, \quad \dot{\varrho}_3 = -\gamma_3 z_3 \alpha_3,$$

Nonlinear control

Backstepping with tuning functions

$$z_1 = x_1 - y, \quad z_2 = x_2 - \alpha_1$$
$$z_3 = x_3 - \alpha_2, \quad u = \varrho_3 \alpha_3$$

$$\alpha_1 = -c_1 z_1$$

$$\begin{aligned}\alpha_3 = &-\kappa_3 z_3 x_2^2 - \vartheta_5 x_2 + \partial_{x_1}\{\alpha_2\} x_2 \\ &- (x_1^4 + x_1^2 \\ &+ x_2^2 + x_3^2 + 1)\kappa_3 z_3 \partial_{x_2}\{\alpha_2\}^2 \\ &+ \partial_\vartheta\{\alpha_3\}\dot{\vartheta} - x_3\vartheta_6 - \beta_2 z_2 \\ &- \kappa_3 z_2^2 z_3 - c_3 z_3 - x_3^2 \kappa_3 z_3 \\ &+ (\vartheta_1 + x_1(\vartheta_2 + x_1\vartheta_3) \\ &+ x_2\vartheta_4 + x_3(\beta_2 + 2\kappa_3 z_2 z_3))\partial_{x_2}\{\alpha_2\} \\ &+ \dot{\varrho}_2 \partial_{\varrho_2}\{\alpha_2\}\end{aligned}$$

$$\begin{aligned}\alpha_2 = \varrho_2(&-\vartheta_3 x_1^2 - \vartheta_2 x_1 - x_1 \\ &+ y - \vartheta_1 - x_2\vartheta_4 - c_2 z_2 \\ &- (x_1^4 + x_1^2 + x_2^2 + 1)\kappa_2 z_2 \\ &+ x_2 \partial_{x_1}\{\alpha_1\})\end{aligned}$$

$$\begin{aligned}\dot{\vartheta} = \Gamma[&z_2 - z_3\partial_{x_2}\{\alpha_2\}, \\ &x_1(z_2 - z_3\partial_{x_2}\{\alpha_2\}), \\ &x_1^2(z_2 - z_3\partial_{x_2}\{\alpha_2\}), \\ &x_2(z_2 - z_3\partial_{x_2}\{\alpha_2\}), \\ &x_2 z_3, x_3 z_3]^{\mathrm{T}},\end{aligned}$$

$$\dot{\beta}_2 = -\gamma_2 z_3(-x_2 + \alpha_1 + x_3\partial_{x_2}\{\alpha_2\}),$$
$$\dot{\varrho}_2 = -\gamma_2 z_2 \alpha_2/\varrho_2, \quad \dot{\varrho}_3 = -\gamma_3 z_3 \alpha_3,$$

Nonlinear control

Mathematica to the rescue...

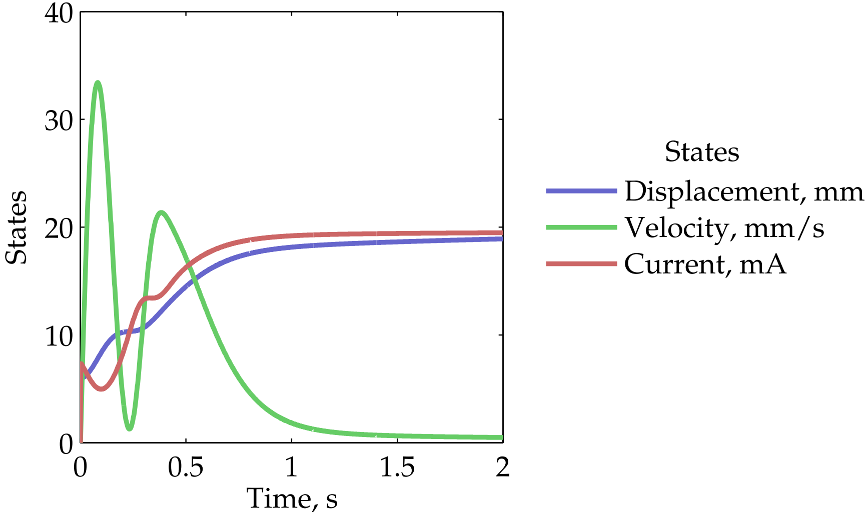
Mathematica is ideal for this task:

1. Control algebra calculated,
2. Embedded Matlab (or C) code written,
3. Output LaTeX typesetting code.

This is all *automatic*, and for a large class of nonlinear systems.

But does it work?

Simulation results only at this stage. . .



The end

1. Zero stiffness is a good idea for vibration isolation…
2. …but impossible because it's unstable

3. Control can solve that problem…
4. …but there's more work to be done

Thanks

It's been a great conference!

- Email:
  `will@mecheng.adelaide.edu.au`
- Slides available at:
  `http://www.mecheng.adelaide.edu.au/~will/`